AD-775 247

DREWS INSTITUTIONALIZED DIVVY ECONOMY

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Prepared for:

Office of Naval Research
Atomic Energy Commission
National Science Foundation

September 1973

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Department of Operations Research Stanford University	2a. REPORT SECURITY CLASSIFICATION AD-775 247 2b. GROUP

3. REPORT TITLE

DREWS DIVVY ECONOMY

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (Last name, first name, initial)

DANTZIG, George B.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1973	~~5~~ 9	0

8a. CONTRACT OR GRANT NO. N00014-67-A-0112-0011 b. PROJECT NO. NR-047-064 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 73-7 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Operations Research Program Code 434 Office of Naval Research Washington, D.C. 20360

13. ABSTRACT

This is a simplified version of an economy considered by W. P. Drews, in which the sizes of the institutionalized "consumer groups" and the prices charged by other institutions controlling "resources" are manipulated by these institutions in an effort for each to achieve its share of the total money flows as agreed upon by the "political" process (for example by traditions and negotiations).

This institutionalized view of the economy injects into the usual framework of technological relations an additional mechanism (which Drews calls the invisible hand), the "political" process, which can arbitrarily set the proportions of total money flows to different institutions. Our purpose is to show that once these are agreed upon, all other quantities, such as the levels of industrial production, prices of consumer goods and resources, and the sizes of consumer groups can be determined. Brouwer's Fixed-Point theorem is applied to prove the last statement.

DD FORM 1473 1 JAN 64

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Security Classification

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
INSTITUTIONALIZED ECONOMY						
LEONTIEF SYSTEM						
MATRIX RESCALING						
PRICE DETERMINATION						
DIVVY ECONOMY						
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by

George B. Dantzig

TECHNICAL REPORT 73-7

September 1973

DEPARTMENT OF OPERATIONS RESEARCH

Stanford University
Stanford, California

Reproduction and distribution only, of this report was partially supported by the Office of Naval Research under contract N-00014-67-A-0112-C011; U.S. Atomic Energy Commission Contract AT(04-3)-326-PA #18; and National Science Foundation Grant GP 31393X1.

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by

George B. Dantzig

In this variant of an economy considered by W. P. Drews there are r resource groups each of which sells their basic resource (e.g., labor, oil, coal) to n industries (activities) that produce consumer goods. By adjusting the (relative) prices $(\lambda_1, \lambda_2, \ldots, \lambda_r)$ that they charge for their resources, they can alter the proportions $(\gamma_1, \gamma_2, \ldots, \gamma_r)$ of total money flows they recieve relative to one another. By tradition, the political process, or by arbitration the proportions γ_i are given. Ownership of the resource groups is in the hands of s consumer groups each of whom may own wholly or part of a resource group. For our purposes the money flows $(\gamma_1, \gamma_2, \ldots, \gamma_r)$ are transferred in some known way to the consumer groups and result in $(\delta_1, \delta_2, \ldots, \delta_s)$ being the known given relative money flows to the s consumer groups. The cost of resource inputs (which includes the cost of labor), however, can affect the prices of n types of consumer goods and therefore can affect the total cost that each consumer group must pay-out to buy their characteristic bill-of-goods. If the latter cost is too high a consumer group will attempt to alter (reduce) the proportions of the population $(\mu_1, \mu_2, \ldots, \mu_s)$ aligned with it. For any selected set of resource prices $(\lambda_1, \ldots, \lambda_r)$, it may not be possible, however, for all consumer groups to adjust their sizes μ_j

simultaneously to acheive an <u>exact</u> <u>balance</u> between the revenues each receives and each pays out to purchase consumer goods. Our purpose will be to show, however, that we can "divvy" up the economy according to any preassigned money flow amounts $(\gamma_1, \gamma_2, \ldots, \gamma_r)$, $(\delta_1, \delta_2, \ldots, \delta_s)$ and can find prices $(\lambda_1, \lambda_2, \ldots, \lambda_r)$ and sizes $(\mu_1, \mu_2, \ldots, \mu_s)$ so that the implied cost of the consumer goods for each consumer group is in exact balance with its revenues.

The s-consumer groups are assumed to have characteristic consumption vectors $C_1, C_2, \ldots, C_s$ of n types of consumer goods expressed in absolute terms per person ($C_j \geq 0$ are column vectors). The economy will be assumed to consist of n activities that produce n types of consumer goods inter-related by a square Leontief type input-output matrix L. The level of activities X thus satisfy:

$$(1) \qquad LX = \sum_{j=1}^{s} C_j \mu_j = C\mu \quad , \qquad \sum_{j=1}^{s} \mu_j = 1, \; \mu_j \geq 0 \; .$$

We now assume that the kth consumer good activity must purchase (per unit of activity level) $R_{ik} \geq 0$ units of basic resources i. Thus the total cost of purchases of all basic resources per unit of activity k is $\sum_{i=1}^{r} \lambda_i R_{ik}$. The row vector of costs of all n activities per unit level is $\sum_{i=1}^{r} \lambda_i R_i$ where $R_i = (R_{i1}, R_{i2}, \ldots, R_{in})$. The implicit prices for consumer goods Y thus satisfy:

(2) $$YL = \sum_{i=1}^{r} \lambda_i R_i = \lambda R \quad , \qquad \sum_{i=1}^{r} \lambda_i = 1, \ \lambda_i \geq 0 \ .$$

The various relations may be usefully displayed in tableau form:

	Consumer-goods production levels X	Consumer group size μ	
Consumer Goods: Y (implicit prices)	Leontief square input-output matrix: L = =	Characteristic bill of goods: C	
Resource Prices: λ	R → Resource Inputs	$\ell\gamma$ ↗ $\ell\delta$	Money flows

where ℓ is the scalar proportionality factor. Thus from our definitions the revenues received for the i-th resource must satisfy for some choice of scalar factor ℓ

(3) $\lambda_i \ R_i X = \ell\gamma_i$ for $i = (1, \ldots, r)$

and the expenditures for consumer goods by the j-th consumer group must satisfy for the same choice of scalar factor ℓ

(4) $YC_j \ \mu_j = \ell\delta_j$ for $j = (1, \ldots, s)$

where the equality of the scalar factors can be shown from (1) and (2). Substituting the values of X and Y from (1) and (2) we have

(5) $\lambda_i \ R_i \ L^{-1} \ C\mu = \ell\gamma_i$ for $i = (1, \ldots, r)$

(6) $\lambda R \ L^{-1} \ C\mu = \ell\delta_j$ for $j = (1, \ldots, s)$

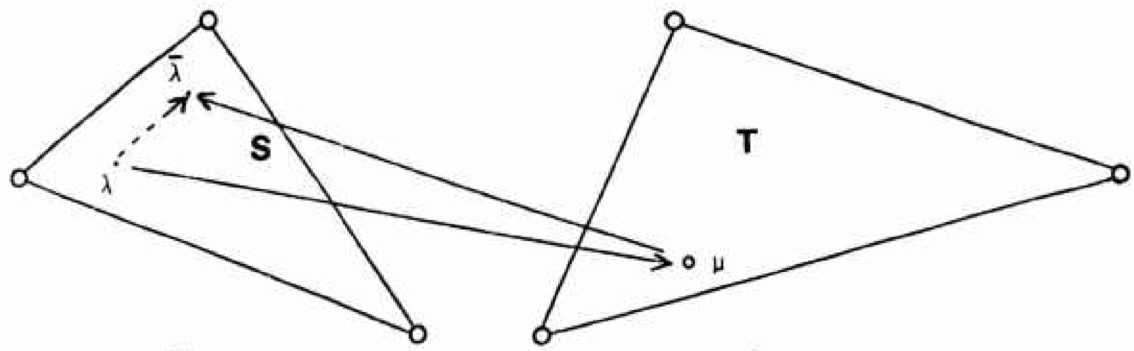
If we set

$$[M_{ij}] = [R_i \, L^{-1} \, C_j] \qquad \text{where } M \text{ is } r \times s \, ,$$

then (5) and (6) simply state that we seek a λ in the simplex $S = \{\lambda | \lambda_i \geq 0, \sum_{i=1}^{r} \lambda_i = 1\}$ and a μ in the simplex $T = \{\mu | \mu_j \geq 0, \sum_{j=1}^{s} \mu_j = 1\}$ such that the <u>rescaled</u> <u>matrix</u> $[\lambda_i \, M_{ij} \, \mu_j]$ has row sums proportional to $\gamma = (\gamma_1, \ldots, \gamma_r)$ and column sums proportional to $\delta = (\delta_1, \ldots, \delta_s)$.

<u>Theorem</u>: Given $M > 0$ and $\gamma = (\gamma_1, \ldots, \gamma_r) \geq 0, \delta = (\delta_1, \ldots, \delta_s) \geq 0$, $\Sigma\gamma_i = \Sigma\delta_j$, then there exist $\lambda \in S$, $\mu \in T$ and a scalar ℓ such that $\sum_i \lambda_i M_{ij} \mu_j = \ell\delta_j, \sum_j \lambda_i M_{ij} \mu_j = \ell\gamma_i$ for $i = (1, \ldots, r)$ and $j = (1, \ldots, s)$.

<u>Proof</u>: Starting with any $\lambda \in S$ determine a mapping of $\lambda \to \mu \in T$ by setting $\mu_j' = \delta_j / \sum_{i=1}^{s} \lambda_i M_{ij}$ and $\mu_j = \mu_j' / \sum_{j=1}^{s} \mu_j'$ for $j = 1, \ldots, s$.



<u>S Simplex</u>: $\{\lambda | \sum_{i=1}^{r} \lambda_i = 1, \lambda_i \geq 0\}$ <u>T Simplex</u>: $\{\mu | \sum_{j=1}^{s} \mu_j = 1, \mu_j \geq 0\}$

Next map bakc this $\mu \to \bar{\lambda} \in S$ by $\bar{\lambda}' = \gamma_i / \sum_{j=1}^{s} M_{ij}\, \mu_j$ and set $\bar{\lambda}_i' = \bar{\lambda}_i' / \sum_{i=1}^{r} \lambda_i'$ for $i = 1, \ldots, r$. The composite of the two successive mappings is a mapping in S: $\lambda \to \bar{\lambda}$ which is clearly continuous in λ if $M_{ij} > 0$. By the Brouwer Fixed-Point Theorem, there exists a λ such that $\bar{\lambda} = \lambda$. Q.E.D.